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RESYNCHRONIZATION FOR MULTIPROCESSOR DSP IMPLEMENTATION — PART 1: MAXIMUM THROUGHPUT RESYNCHRONIZATION[1]

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1. Abstract

This paper introduces a technique, called *resynchronization*, for reducing synchronization overhead in multiprocessor implementations of digital signal processing (DSP) systems. The technique applies to arbitrary collections of dedicated, programmable or configurable processors, such as combinations of programmable DSPs, ASICS, and FPGA subsystems. Thus, it is particularly well suited to the evolving trend towards heterogeneous single-chip multiprocessors in DSP systems. Resynchronization exploits the well-known observation [36] that in a given multiprocessor implementation, certain synchronization operations may be *redundant* in the sense that their associated sequencing requirements are ensured by other synchronizations in the system. The goal of resynchronization is to introduce new synchronizations in such a way that the number of additional synchronizations that become redundant exceeds the number of new synchronizations that are added, and thus the net synchronization cost is reduced.

Our study is based in the context of *self-timed* execution of *iterative dataflow* specifications of digital signal processing applications. An iterative dataflow specification consists of a dataflow representation of the body of a loop that is to be iterated infinitely; dataflow programming in this form has been employed extensively, particularly in the context of software and system-level design for digital signal processing applications. Self-timed execution refers to a

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combined compile-time/run-time scheduling strategy in which processors synchronize with one another only based on inter-processor communication requirements, and thus, synchronization of processors at the end of each loop iteration does not generally occur.

After reviewing our model for the analysis of synchronization overhead, we define the general form of our resynchronization problem; we show that optimal resynchronization is intractable by establishing a correspondence to the *set covering* problem; and based on this correspondence, we develop an efficient heuristic for resynchronization. Also, we show that for a broad class of iterative dataflow graphs, optimal resynchronizations can be computed by means of an efficient polynomial-time algorithm. We demonstrate the utility of our resynchronization techniques through a practical example of a music synthesis system.

2. Introduction

This paper is concerned with implementation of iterative, dataflow-dominated algorithms on embedded multiprocessor systems. In the DSP domain, such multiprocessors typically consist of one or more CPU's (micro-controllers or programmable digital signal processors), and one or more application-specific hardware components (implemented as custom ASICs or on reconfigurable logic such as FPGAs). Such embedded multiprocessor systems are becoming increasingly common today in applications ranging from digital audio/video equipment to portable devices such as cellular phones and PDA's. A digital cellular phone, for example, typically consists of a micro-controller, a DSP, and custom ASIC circuitry. With increasing levels of integration, it is now feasible to integrate such heterogeneous systems entirely on a single chip. The design task of such multiprocessor systems-on-a-chip is complex, and the complexity will only increase in the future.

One of the critical issues in the design of embedded multiprocessors is managing communication and synchronization between the heterogeneous processing elements. In this paper, we focus on the problem of minimizing communication and synchronization overhead in embedded multiprocessors. We propose algorithms that automate the process of designing synchronization points in a shared-memory multiprocessor system with the objective of reducing synchronization overhead.

Specifically, we develop a technique called *resynchronization* for reducing the rate at which synchronization operations must be performed in a shared-memory multiprocessor system. Resynchronization is based on the concept that there can be redundancy in the synchronization functions of a given multiprocessor implementation [36]. Such redundancy arises whenever the objective of one synchronization operation is guaranteed as a side effect of other synchronizations in the system. In the context of noniterative execution, Shaffer showed that the amount of run-time overhead required for synchronization can be reduced significantly by detecting redundant synchronizations and implementing only those synchronizations that are found not to be redundant; an efficient, optimal algorithm was also proposed for this purpose [36]; and this algorithm was subsequently extended to handle iterative computations [5]. The objective of resynchronization is to introduce new synchronizations in such a way that the number of original synchronizations that consequently become redundant is significantly more than number of new synchronizations.

2.1 Iterative synchronous dataflow

We study this problem in the context of self-timed execution of iterative *synchronous dataflow* (SDF) specifications. An iterative SDF specification consists of an SDF representation of a computation that is to be iterated infinitely. In SDF, an application is represented as a directed graph in which vertices (**actors**) represent computational tasks, edges specify data dependences, and the number of data values (**tokens**) produced and consumed by each actor is fixed. This form of “synchrony” should not be confused with the use of “synchronous” in synchronous languages [3]. The task represented by an actor can be of arbitrary complexity. In DSP design environments, it typically ranges in complexity from a basic operation such as addition or subtraction to a signal processing subsystem such as an FFT unit or an adaptive filter.

Although the model is too restricted for many general-purpose applications, iterative SDF has proven to be a useful framework for representing a significant class of digital signal processing (DSP) algorithms, and it has been used as the foundation for numerous DSP design environments, in which applications are represented as hierarchies of block diagrams. Examples of commercial tools that employ SDF are the Signal Processing Worksystem (SPW) by Cadence, COSSAP by Synopsys, and HP Ptolemy by Hewlett-Packard. Research tools developed at universities that use SDF and related models include DESCARTES [34], GRAPE [21], Ptolemy [8], and the Warp compiler [32]. A wide variety of techniques have been developed to schedule SDF specifications for efficient multiprocessor implementation, such as those described in [1, 2, 9, 14, 15, 25, 29, 32, 37, 39]. The techniques developed in this paper can be used as a post-processing step to improve the performance of implementations that use any of these scheduling techniques.

Each SDF edge has associated a non-negative integer *delay*. SDF delays represent initial tokens, and specify dependencies between iterations of actors in iterative execution. For example, if tokens produced by the kth invocation of actor A are consumed by the $(k + 2)$th invocation of actor B, then the edge (A, B) contains two delays. We assume that the input SDF graph is *homogeneous*, which means that the numbers of tokens produced and consumed are identically unity. However, since efficient techniques have been developed to convert general SDF graphs into homogeneous graphs [23], our techniques can easily be adapted to general SDF graphs. We refer to a homogeneous SDF graph as a **dataflow graph** (**DFG**).

2.2 Self-timed Scheduling Model

Our implementation model involves a *self-timed* scheduling strategy [24]. Each processor executes the tasks assigned to it in a fixed order that is specified at compile time (i. e. statically). Before firing an actor, a processor waits for the data needed by that actor to become available. Thus, processors are required to perform run-time synchronization when they communicate data. This provides robustness when the execution times of tasks are not known precisely or when they may exhibit occasional deviations from their compile-time estimates.

Such a self-timed strategy is well-suited for implementation of signal processing and communication systems owing to the dataflow nature of the computations involved. Examples of such systems are high speed modems, image compression and decompression systems, and wireless communications. The data flow between components in such systems (e.g. between the channel equalizer and the Viterbi decoder in a high speed modem) tends to be regular and predictable. Thus communication between processing elements implementing these different components, e.g. a DSP implementing the equalizer and a dedicated ASIC implementing the Viterbi decoder, will also be predictable, allowing for a self-timed implementation. The key motivation behind such an implementation is that no run-time scheduling of tasks is required; this considerably reduces communication and synchronization overhead. In other words, the predictable dataflow in these applications is leveraged by employing a self-timed strategy to yield highly optimized system implementations.

Interprocessor communication (**IPC**) between processors is assumed to take place through shared memory, which could be global memory between all processors, or it could be distributed between pairs of processors (for example, hardware first-in-first-out (FIFO) queues or dual ported memory). Such simple communication mechanisms, as opposed to cross bars and elaborate interconnection networks, are common in embedded systems, owing to their simplicity and low cost.

Sender-receiver synchronization is performed by setting and testing flags in shared memory; Section 4.2 provides details on the assumed synchronization protocols. Interfaces between hardware and software are typically implemented using memory-mapped registers in the address space of the programmable processor, which can be viewed as a kind of shared memory. Synchronization of such interfaces is achieved using flags that can be tested and set by the programmable component, and the same can be done by an interface controller on the hardware side [16]. Thus, in our context, effective resynchronization results in a significantly reduced rate of accesses to shared memory for the purpose of synchronization.

The resynchronization techniques developed in this paper are designed to improve the throughput of multiprocessor implementations. Frequently in real-time signal processing systems, latency is also an important issue, and although resynchronization improves the throughput, it generally degrades (increases) the latency. In this paper, we address the problem of resynchronization under the assumption that an arbitrary increase in latency is acceptable. Such a scenario arises when the computations occur in a feedforward manner, e.g audio/video decoding for playback from media such as DVD (Digital Video Disk), and also for a wide variety of simulation applications. The companion paper [7] examines the relationship between resynchronization and latency, and addresses the problem of optimal resynchronization when only a limited increase in latency is tolerable. Such latency constraints are present in interactive applications such as video conferencing and telephony, where beyond a certain point latency becomes annoying to the user. Preliminary versions of the material in this paper and the companion paper have been summarized in [6] and [4], respectively.

3. Background

We represent a DFG by an ordered pair (V, E), where V is the set of vertices (**actors**) and E is the set of edges. We refer to the source and sink actors of a DFG edge e by $src(e)$ and $snk(e)$, we denote the delay on e by $delay(e)$, and we frequently represent e by the ordered pair $(src(e), snk(e))$. We say that e is an **output edge** of $src(e)$, and e is an **input edge** of $snk(e)$. Edge e is **delayless** if $delay(e) = 0$, and it is a **self loop** if $src(e) = snk(e)$.

Given $x, y \in V$, we say that x is a **predecessor** of y if there exists $e \in E$ such that $src(e) = x$ and $snk(e) = y$; we say that x is a **successor** of y if y is a predecessor of x. A **path** in (V, E) is a finite sequence $(e_1, e_2, \ldots, e_n)$, where each e_i is a member of E, and $snk(e_1) = src(e_2)$, $snk(e_2) = src(e_3)$, $\ldots$, $snk(e_{n-1}) = src(e_n)$. We say that the path $p = (e_1, e_2, \ldots, e_n)$ **contains** each e_i and each contiguous subsequence of $(e_1, e_2, \ldots, e_n)$; p is **directed from** $src(e_1)$ **to** $snk(e_n)$; and each member of

$$\{src(e_1), src(e_2), \ldots, src(e_n), snk(e_n)\}$$

is **traversed by** p. A path that is directed from some vertex to itself is called a **cycle**, and a **simple cycle** is a cycle of which no proper subsequence is a cycle.

If $(p_1, p_2, \ldots, p_k)$ is a finite sequence of paths such that $p_i = (e_{i,1}, e_{i,2}, \ldots, e_{i,n_i})$, for

$1 \le i \le k$, and $snk(e_{i, n_i}) = src(e_{i+1, 1})$, for $1 \le i \le (k-1)$, then we define the **concatenation** of $(p_1, p_2, \ldots, p_k)$, denoted $\langle (p_1, p_2, \ldots, p_k) \rangle$, by

$$\langle (p_1, p_2, \ldots, p_k) \rangle \equiv (e_{1,1}, \ldots, e_{1, n_1}, e_{2,1}, \ldots, e_{2, n_2}, \ldots, e_{k,1}, \ldots, e_{k, n_k}) .$$

Clearly, $\langle (p_1, p_2, \ldots, p_k) \rangle$ is a path from $src(e_{1,1})$ to $snk(e_{k, n_k})$.

If $p = (e_1, e_2, \ldots, e_n)$ is a path in a DFG, then we define the **path delay** of p, denoted $Delay(p)$, by

$$Delay(p) = \sum_{i=1}^{n} delay(e_i). \tag{1}$$

Since the delays on all DFG edges are restricted to be non-negative, it is easily seen that between any two vertices $x, y \in V$, either there is no path directed from x to y, or there exists a (not necessarily unique) **minimum-delay path** between x and y. Given a DFG G, and vertices x, y in G, we define $\rho_G(x, y)$ to be equal to ∞ if there is no path from x to y, and equal to the path delay of a minimum-delay path from x to y if there exist one or more paths from x to y. If G is understood, then we may drop the subscript and simply write "ρ" in place of "ρ_G". It is easily seen that minimum delay path lengths satisfy the following *triangle inequality*

$$\rho_G(x, z) + \rho_G(z, y) \ge \rho_G(x, y), \text{ for any } x, y, z \text{ in } G. \tag{2}$$

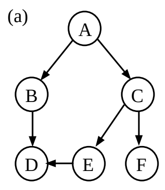
By a **subgraph** of (V, E), we mean the directed graph formed by any $V' \subseteq V$ together with the set of edges $\{e \in E \mid src(e), snk(e) \in V'\}$. We denote the subgraph associated with the vertex-subset V' by $subgraph(V')$. We say that (V, E) is **strongly connected** if for each pair of distinct vertices x, y, there is a path directed from x to y and there is a path directed from y to x. We say that a subset $V' \subseteq V$ is strongly connected if $subgraph(V')$ is strongly connected. A **strongly connected component (SCC)** of (V, E) is a strongly connected subset $V' \subseteq V$ such that no strongly connected subset of V properly contains V'. If V' is an SCC, then when there is no ambiguity, we may also say that $subgraph(V')$ is an SCC. If C_1 and C_2 are distinct SCCs in (V, E), we say that C_1 is a **predecessor SCC** of C_2 if there is an edge directed from some vertex in C_1 to some vertex in C_2; C_1 is a **successor SCC** of C_2 if C_2 is a predecessor SCC of C_1. An SCC is a **source SCC** if it has no predecessor SCC; an SCC is a **sink SCC** if it has no successor SCC; and an SCC is an **internal SCC** if it is neither a source SCC nor a sink SCC. An edge is a **feedforward** edge if it is not contained in an SCC, or equivalently, if it is not contained in a cycle; an edge that is contained in at least one cycle is called a **feedback** edge.

We denote the number of elements in a finite set S by $|S|$.

4. Synchronization model

In this section, we review the model that we use for analyzing synchronization in self-timed multiprocessor systems. The model was originally developed in [38] to study the execution patterns of actors under self-timed evolution, and in [5], the model was augmented for the analysis of synchronization overhead.

A DFG representation of an application is called an **application DFG**. For each task v in a given application DFG G, we assume that an estimate $t(v)$ (a positive integer) of the execution time is available. Given a multiprocessor schedule for G, we derive a data structure called the **IPC graph**, denoted G_{ipc}, by instantiating a vertex for each task, connecting an edge from each task to the task that succeeds it on the same processor, and adding an edge that has unit delay from the last task on each processor to the first task on the same processor. Also, for each edge (x, y) in G that connects tasks that execute on different processors, an *IPC edge* is instantiated in G_{ipc} from x to y. Figure 1(c) shows the IPC graph that corresponds to the application DFG of Figure 1(a) and the processor assignment / actor ordering of Figure 1(b).



(b)

Processor	Actor ordering
Proc. 1	B, D, F
Proc. 2	A, C, E

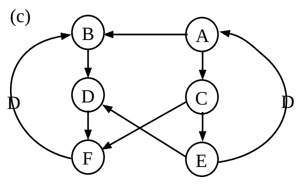


Figure 1. Part (c) shows the IPC graph that corresponds to the DFG of part (a) and the processor assignment / actor ordering of part (b). A “D” on top of an edge represents a unit delay.

Each edge (v_j, v_i) in G_{ipc} represents the **synchronization constraint**

$$start(v_i, k) \geq end(v_j, k - delay((v_j, v_i))), \tag{3}$$

where $start(v, k)$ and $end(v, k)$ respectively represent the time at which invocation k of actor v begins execution and completes execution.

4.1 The synchronization graph

Initially, an IPC edge in G_{ipc} represents two functions: reading and writing of tokens into the corresponding buffer, and synchronization between the sender and the receiver. To differentiate these functions, we define another graph called the **synchronization graph**, in which edges between tasks assigned to different processors, called **synchronization edges**, represent *synchronization constraints only.*

Initially, the synchronization graph is identical to G_{ipc}. However, resynchronization modifies the synchronization graph by adding and deleting synchronization edges. After resynchronization, the IPC edges in G_{ipc} represent buffer activity, and are implemented as buffers in shared memory, whereas the synchronization edges represent synchronization constraints, and are implemented by updating and testing flags in shared memory. If there is an IPC edge as well as a synchronization edge between the same pair of actors, then the synchronization protocol is executed before the buffer corresponding to the IPC edge is accessed to ensure sender-receiver synchronization. On the other hand, if there is an IPC edge between two actors in the IPC graph, but there is no synchronization edge between the two, then no synchronization needs to be done before accessing the shared buffer. If there is a synchronization edge between two actors but no IPC edge, then no shared buffer is allocated between the two actors; only the corresponding synchronization protocol is invoked.

4.2 Synchronization protocols

Given a synchronization graph (V, E), and a synchronization edge $e \in E$, if e is a feedforward edge then we apply a synchronization protocol called **feedforward synchronization**

(FFS), which guarantees that $snk(e)$ never attempts to read data from an empty buffer (to prevent underflow), and $src(e)$ never attempts to write data into the buffer unless the number of tokens already in the buffer is less than some pre-specified limit, which is the amount of memory allocated to that buffer (to prevent overflow). This involves maintaining a count of the number of tokens currently in the buffer in a shared memory location. This count must be examined and updated by each invocation of $src(e)$ and $snk(e)$.

If e is a feedback edge, then we use a more efficient protocol, called **feedback synchronization (FBS)**, that only explicitly ensures that underflow does not occur. Such a simplified protocol is possible because each feedback edge has a buffer requirement that is bounded by a constant, called the **self-timed buffer bound** of the edge, which can be computed efficiently from the synchronization graph topology [5]. In this protocol, we allocate a shared memory buffer of size equal to the self-timed buffer bound of e, and rather than maintaining the token count in shared memory, we maintain a copy of the *write pointer* into the buffer (of the source actor). After each invocation of $src(e)$, the write pointer is updated locally (on the processor that executes $src(e)$), and the new value is written to shared memory. It is easily verified that to prevent underflow, it suffices to block each invocation of the sink actor until the *read pointer* (maintained locally on the processor that executes $snk(e)$) is found to be not equal to the current value of the write pointer. For a more detailed discussion of the FFS and FBS protocols, the reader is referred to [5].

An important parameter in an implementation of FFS or FBS is the **back-off time** T_b. If a receiving processor finds that the corresponding IPC buffer is full, then the processor releases the shared memory bus, and waits T_b time units before requesting the bus again to re-check the shared memory synchronization variable. Similarly, a sending processor waits T_b time units between successive accesses of the same synchronization variable. The back-off time can be selected experimentally by simulating the execution of the given synchronization graph (with the available execution time estimates) over a wide range of candidate back-off times, and selecting the back-off time that yields the highest simulated throughput.

4.3 Estimated throughput

If the execution time of each actor v is a fixed constant $t^*(v)$ for all invocations of v, and the time required for IPC is ignored (assumed to be zero), then as a consequence of Reiter's analysis in [33], the throughput (number of DFG iterations per unit time) of a synchronization graph G is given by $1/(\lambda_{max}(G))$, where

$$\lambda_{max}(G) \equiv \max_{\text{cycle } C \text{ in } G} \left\{ \frac{\sum_{v \in C} t^*(v)}{Delay(C)} \right\}. \tag{4}$$

If the maximum in (4) is infinite, there exists at least one delay free cycle in G, which means that the schedule modeled by the synchronization graph is deadlocked. In the remainder of this paper, we are concerned only with synchronization graphs that result from schedules that are not deadlocked. Thus, we assume the absence of delay-free cycles. In practice, this assumption is not a problem since delay-free cycles can be detected efficiently [18].

The quotient in (3) is called the **cycle mean** of the cycle C, and the entire quantity on the RHS of (3) is called the **maximum cycle mean** of G. A cycle in G whose cycle mean is equal to the maximum cycle mean of G is called a **critical cycle** of G. Since in our problem context, we only have execution time estimates available instead of exact values, we replace $t^*(v)$ with the corresponding estimate $t(v)$ in (3) to obtain an estimate of the maximum cycle mean. The reciprocal of this estimate of the maximum cycle mean is called the **estimated throughput**. The objective of resynchronization is to increase the *actual throughput* by reducing the rate at which synchronization operations must be performed, while making sure that the estimated throughput is not degraded.

4.4 Preservation of synchronization graphs

Any transformation that we perform on the synchronization graph must respect the synchronization constraints implied by G_{ipc}. If we ensure this, then we only need to implement the synchronization edges of the optimized synchronization graph. If $G_1 = (V, E_1)$ and

$G_2 = (V, E_2)$ are synchronization graphs with the same vertex-set and the same set of intraprocessor edges (edges that are not synchronization edges), we say that G_1 **preserves** G_2 if for all $e \in E_2$ such that $e \notin E_1$, we have $\rho_{G_1}(src(e), snk(e)) \le delay(e)$.

The following theorem, which is developed in [5], underlies the validity of our synchronization optimizations.

Theorem 1: The synchronization constraints (as specified by (3)) of G_1 imply the constraints of G_2 if G_1 preserves G_2.

Intuitively, Theorem 1 is true because if G_1 preserves G_2, then for every synchronization edge e in G_2, there is a path in G_1 that enforces the synchronization constraint specified by e.

Definition 1: A synchronization edge is **redundant** in a synchronization graph G if its removal yields a graph that preserves G. The synchronization graph G is **reduced** if G contains no redundant synchronization edges.

For example, in Figure 1(c), the synchronization edge (C, F) is redundant due to the path $((C, E), (E, D), (D, F))$.

In [5], it is shown that if all redundant edges in a synchronization graph are removed, then the resulting graph preserves the original synchronization graph.

From Definition 1, we have the following fact concerning redundant synchronization edges.

Fact 1: Suppose that $G = (V, E)$ is a synchronization graph and s is a redundant synchronization edge in G. Then there exists a simple path p in G directed from $src(s)$ to $snk(s)$ such that p does not contain s, and $Delay(p) \le delay(s)$.

Proof: Let $G' \equiv (V, (E - \{s\}))$ denote the synchronization graph that results when we remove s from G. Then from Definition 1, there exists a path p' in G' directed from $src(s)$ to $snk(s)$ such that

$$Delay(p') \le delay(s). \tag{5}$$

Now observe that every edge in G' is also contained in G, and thus, G contains the path p'. If p' is a simple path, then we are done. Otherwise, p' can be expressed as a concatenation

$$\langle (q_0, C_1, q_1, C_2, \ldots, q_{n-1}, C_n, q_n) \rangle \text{, } n \geq 1 \text{,} \tag{6}$$

where each q_i is a simple path, at least one q_i is non-empty, and each C_j is a (not necessarily simple) cycle. Since valid synchronization graphs cannot contain delay-free-cycles (Section 4.3), we must have $Delay(C_k) \geq 1$ for $1 \leq k \leq n$. Thus, since each C_i originates and terminates at the same actor, the path $p'' = \langle q_0, q_1, \ldots, q_n \rangle$ is a simple path directed from $src(s)$ to $snk(s)$ such that $Delay(p'') < Delay(p')$. Combining this last inequality with (5) yields

$$Delay(p'') < delay(s) \text{.} \tag{7}$$

Furthermore, since p' is contained in G, it follows from the construction of p'', that p'' must also be contained in G.

Finally, since p' is contained in G', G' does not contain s, and the set of edges contained in p'' is a subset of the set of edges contained in p', we have that p'' does not contain s. *QED*.

5. Related work

Shaffer has developed an algorithm that removes redundant synchronizations in the self-timed execution of a non-iterative DFG [36]. This technique was subsequently extended to handle iterative execution and DFG edges that have delay [5]. These approaches differ from the techniques of this paper in that they only consider the redundancy induced by the *original* synchronizations; they do not consider the addition of new synchronizations.

Filo, Ku and De Micheli have studied synchronization rearrangement in the context of minimizing the controller area for hardware synthesis of synchronization digital circuitry [11, 12], and significant differences in the underlying analytical models prevent these techniques from applying to our context. In the graphical hardware model of [12], called the *constraint graph* model, each vertex corresponds to a separate hardware device and edges have arbitrary weights that specify sequencing constraints. When the source vertex has bounded execution time, a positive weight $w(e)$ (*forward constraint*) imposes the constraint

$$start(snk(e)) \geq w(e) + start(src(e)),$$

while a negative weight (*backward constraint*) implies

$$start(snk(e)) \leq w(e) + start(src(e)).$$

If the source vertex has unbounded execution time, the forward and backward constraints are relative to the *completion* time of the source vertex. In contrast, in our synchronization graph model, multiple actors can reside on the same processing element (implying zero synchronization cost between them), and the timing constraints always correspond to the case where $w(e)$ is positive and equal to the execution time of $src(e)$.

The implementation models, and associated implementation cost functions are also significantly different. A constraint graph is implemented using a scheduling technique called *relative scheduling* [19], which can roughly be viewed as intermediate between self-timed and fully-static scheduling. In relative scheduling, the constraint graph vertices that have unbounded execution time, called *anchors*, are used as reference points against which all other vertices are scheduled: for each vertex v, an offset f_i is specified for each anchor a_i that affects the activation of v, and v is scheduled to occur once f_i clock cycles have elapsed from the completion of a_i, for each i.

In the implementation of a relative schedule, each anchor has attached control circuitry that generates offset signals, and each vertex has a synchronization circuit that asserts an *activate* signal when all relevant offset signals are present. The resynchronization optimization is driven by a cost function that estimates the total area of the synchronization circuitry, where the offset circuitry area estimate for an anchor is a function of the maximum offset, and the synchronization circuitry estimate for a vertex is a function of the number of offset signals that must be monitored.

As a result of the significant differences in both the scheduling models and the implementation models, the techniques developed for resynchronizing constraint graphs do not extend in any straightforward manner to the resynchronization of synchronization graphs for self-timed multiprocessor implementation, and the solutions that we have developed for synchronization graphs are significantly different in structure from those reported in [12]. For example, the fundamental relationships that we establish between set covering and our use of resynchronization have not emerged in the context of constraint graphs.

6. Resynchronization

We refer to the process of adding one or more new synchronization edges and removing the redundant edges that result as *resynchronization* (defined more precisely below). Figure 2(a) illustrates how this concept can be used to reduce the total number of synchronizations in a multiprocessor implementation. Here, the dashed edges represent synchronization edges. Observe that if we insert the new synchronization edge $d_0(C, H)$, then two of the original synchronization edges — (B, G) and (E, J) — become redundant. Since redundant synchronization edges can be removed from the synchronization graph to yield an equivalent synchronization graph, we see that the net effect of adding the synchronization edge $d_0(C, H)$ is to reduce the number of synchronization edges that need to be implemented by 1. In Figure 2(b), we show the synchronization graph that results from inserting the *resynchronization edge* $d_0(C, H)$ into Figure 2(a), and then removing the redundant synchronization edges that result.

Definition 2 gives a formal definition of resynchronization that we will use throughout the

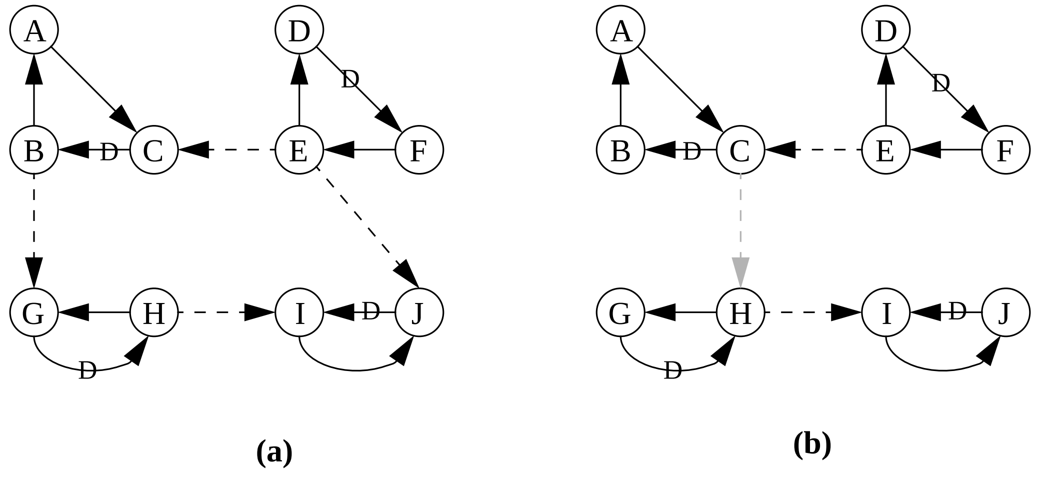


Figure 2. An example of resynchronization.

remainder of this paper. This considers resynchronization only “across” feedforward edges. Resynchronization that includes inserting edges into SCCs, is also possible; however, in general, such resynchronization may increase the estimated throughput (see Theorem 2 at the end of Section 7). Thus, for our objectives, it must be verified that each new synchronization edge introduced in an SCC does not decrease the estimated throughput. To avoid this complication, which requires a check of significant complexity ($O(|V||E|\log_2(|V|))$, where (V, E) is the modified synchronization graph — this is using the Bellman Ford algorithm described in [22]) *for each* candidate resynchronization edge, we focus only on “feedforward” resynchronization in this paper. Future research will address combining the insights developed here for feedforward resynchronization with efficient techniques to estimate the impact that a given *feedback* resynchronization edge has on the estimated throughput.

Opportunities for feedforward resynchronization are particularly abundant in the dedicated hardware implementation of dataflow graphs. If each actor is mapped to a separate piece of hardware, as in the VLSI dataflow arrays of Kung, Lewis, and Lo [20], then for any application graph that is acyclic, every communication channel between two units will have an associated feedforward synchronization edge. Due to increasing circuit integration levels, such isomorphic mapping of dataflow subsystems into hardware is becoming attractive for a growing family of applications. Feedforward synchronization edges often arise naturally in multiprocessor software implementations as well. A software example is presented in detail in Section 10.

Definition 2: Suppose that $G = (V, E)$ is a synchronization graph, and $F \equiv \{e_1, e_2, \ldots, e_n\}$ is the set of all feedforward edges in G. A **resynchronization** of G is a finite set $R \equiv \{e_1', e_2', \ldots, e_m'\}$ of edges that are not necessarily contained in E, but whose source and sink vertices are in V, such that a) $e_1', e_2', \ldots, e_m'$ are feedforward edges in the DFG $G^* \equiv (V, ((E - F) + R))$; and b) G^* preserves G — that is, $\rho_{G^*}(src(e_i), snk(e_i)) \leq delay(e_i)$ for all $i \in \{1, 2, \ldots, n\}$. Each member of R that is not in E is called a **resynchronization edge** of the resynchronization R, G^* is called the **resynchronized graph** associated with R, and this graph is denoted by $\Psi(R, G)$.

If we let G denote the graph in Figure 2, then the set of feedforward edges is $F = \{(B, G), (E, J), (E, C), (H, I)\}$; $R = \{d_0(C, H), (E, C), (H, I)\}$ is a resynchronization of G; Figure 2(b) shows the DFG $G^* = (V, ((E - F) + R))$; and from Figure 2(b), it is easily verified that F, R, and G^* satisfy conditions (a) and (b) of Definition 2.

7. Properties of resynchronization

In this section, we introduce a number of useful properties of resynchronization that we will apply throughout the developments of this paper.

Lemma 1: Suppose that G and G' are synchronization graphs such that G' preserves G, and p is a path in G from actor x to actor y. Then there is a path p' in G' from x to y such that $Delay(p') \le Delay(p)$, and $tr(p) \subseteq tr(p')$, where $tr(\varphi)$ denotes the set of actors traversed by the path φ.

Thus, if a synchronization graph G' preserves another synchronization graph G and p is a path in G from actor x to actor y, then there is at least one path p' in G' such that 1) the path p' is directed from x to y; 2) the cumulative delay on p' does not exceed the cumulative delay on p; and 3) every actor that is traversed by p is also traversed by p' (although p' may traverse one or more actors that are not traversed by p).

For example in Figure 2(a), if we let $x = B$, $y = I$, and $p = ((B, G), (G, H), (H, I))$, then the path $p' = ((B, A), (A, C), (C, H), (H, G), (G, H), (H, I))$ in Figure 2(b) confirms Lemma 1 for this example. Here $tr(p) = \{B, G, H, I\}$ and $tr(p') = \{A, B, C, G, H, I\}$.

Proof of Lemma 1: Let $p = (e_1, e_2, \ldots, e_n)$. By definition of the *preserves* relation, each e_i that is not a synchronization edge in G is contained in G'. For each e_i that is a synchronization edge in G, there must be a path p_i in G' from $src(e_i)$ to $snk(e_i)$ such that $Delay(p_i) \le delay(e_i)$. Let $e_{i_1}, e_{i_2}, \ldots, e_{i_m}$, $i_1 < i_2 < \ldots < i_m$, denote the set of e_is that are synchronization edges in G, and define the path $\tilde{p}$ to be the concatenation

$$\langle (e_1, e_2, \ldots, e_{i_1 - 1}), p_1, (e_{i_1 + 1}, \ldots, e_{i_2 - 1}), p_2, \ldots, (e_{i_{m-1} + 1}, \ldots, e_{i_m - 1}), p_m, (e_{i_m + 1}, \ldots, e_n) \rangle.$$

Clearly, $\tilde{p}$ is a path in G' from x to y, and since $Delay(p_i) \le delay(e_i)$ holds whenever e_i is a synchronization edge, it follows that $Delay(\tilde{p}) \le Delay(p)$. Furthermore, from the construction of $\tilde{p}$, it is apparent that every actor that is traversed by p is also traversed by $\tilde{p}$. *QED.*

The following lemma states that if a resynchronization contains a resynchronization edge e such that there is a delay-free path in the original synchronization graph from the source of e to the sink of e, then e must be redundant in the resychronized graph.

Lemma 2: Suppose that G is a synchronization graph; R is a resynchronization of G; and (x, y) is a resynchronization edge such that $\rho_G(x, y) = 0$. Then (x, y) is redundant in $\Psi(R, G)$. Thus, a minimal resynchronization (fewest number of elements) has the property that $\rho_G(x', y') > 0$ for each resynchronization edge (x', y').

Proof: Let p denote a minimum-delay path from x to y in G. Since (x, y) is a resynchronization edge, (x, y) is not contained in G, and thus, p traverses at least three actors. From Lemma 1, it follows that there is a path p' in $\Psi(R, G)$ from x to y such that

$$Delay(p') = 0, \tag{8}$$

and p' traverses at least three actors. Thus,

$$Delay(p') \le delay((x, y)) \tag{9}$$

and $p' \ne ((x, y))$. Furthermore, p' cannot properly contain (x, y). To see this, observe that if p' contains (x, y) but $p' \ne ((x, y))$, then from (8), it follows that there exists a delay-free cycle in G (that traverses x), and hence that our assumption of a deadlock-free schedule (Section 4.3) is violated. Thus, we conclude that (x, y) is redundant in $\Psi(R, G)$. *QED.*

As a consequence of Lemma 1, the estimated throughput of a given synchronization graph is always less than or equal to that of every synchronization graph that it preserves.

Theorem 2: If G is a synchronization graph, and G' is a synchronization graph that preserves G, then $\lambda_{max}(G') \ge \lambda_{max}(G)$.

Proof: Suppose that C is a critical cycle in G. Lemma 1 guarantees that there is a cycle C' in G' such that a) $Delay(C') \le Delay(C)$, and b) the set of actors that are traversed by C is a subset of the set of actors traversed by C'. Now clearly, b) implies that

$$\sum_{v \text{ is traversed by } C'} t(v) \ge \sum_{v \text{ is traversed by } C} t(v), \tag{10}$$

and this observation together with a) implies that the cycle mean of C' is greater than or equal to the cycle mean of C. Since C is a critical cycle in G, it follows that $\lambda_{max}(G') \ge \lambda_{max}(G)$. *QED.*

Thus, any saving in synchronization cost obtained by rearranging synchronization edges may come at the expense of a decrease in estimated throughput. As implied by Definition 2, we avoid this complication by restricting our attention to feedforward synchronization edges. Clearly, resynchronization that rearranges only feedforward synchronization edges cannot decrease the estimated throughput since no new cycles are introduced and no existing cycles are altered. Thus, with the form of resynchronization that we address in this paper, any decrease in synchronization cost that we obtain is not diminished by a degradation of the estimated throughput.

8. Relationship to set covering

We refer to the problem of finding a resynchronization with the fewest number of elements as the **resynchronization problem**. In Section 9, we formally show that the resynchronization problem is NP-hard, which means that it is unlikely that efficient algorithms can be devised to solve the problem exactly, and thus, that for practical use, we should search for good heuristic solutions [13]. In this section, we explain the intuition behind this result. To establish the NP-hardness of the resynchronization problem, we examine a special case that occurs when there are exactly two SCCs, which we call the **pairwise resynchronization problem**, and we derive a polynomial-time reduction from the classic *set covering problem* [10], a well-known NP-hard problem, to the pairwise resynchronization problem. In the set covering problem, one is given a finite set X and a family T of subsets of X, and asked to find a minimal (fewest number of members) subfamily $T_s \subseteq T$ such that $\bigcup_{t \in T_s} t = X$. A subfamily of T is said to *cover* X if each member of X is contained in some member of the subfamily. Thus, the set covering problem is the problem of finding a minimal cover.

Definition 3: Given a synchronization graph G, let (x_1, x_2) be a synchronization edge in G, and let (y_1, y_2) be an ordered pair of actors in G. We say that (y_1, y_2) **subsumes** (x_1, x_2) in G if $\rho(x_1, y_1) + \rho(y_2, x_2) \le delay((x_1, x_2))$.

Thus, every synchronization edge subsumes itself, and intuitively, if (x_1, x_2) is a synchronization edge, then (y_1, y_2) subsumes (x_1, x_2) if and only if a zero-delay synchronization edge directed from y_1 to y_2 makes (x_1, x_2) redundant.

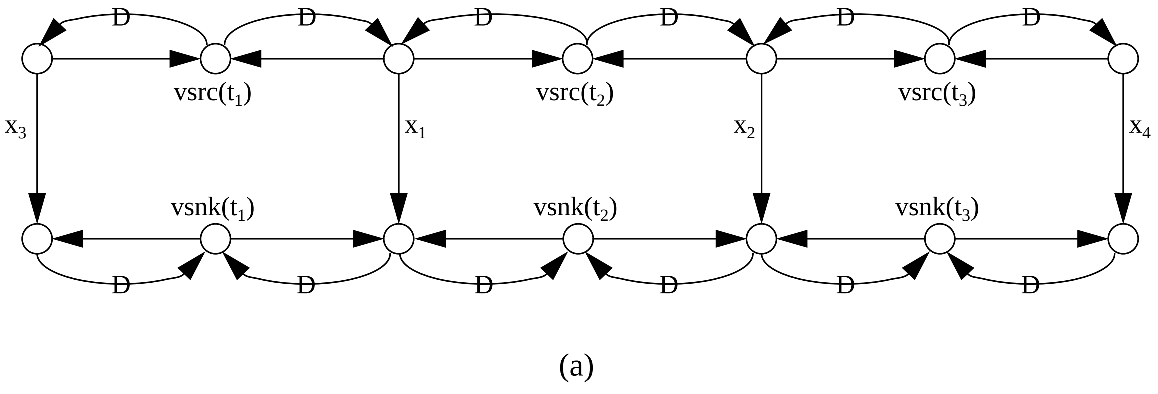
The following fact is easily verified from Definitions 2 and 3.

Fact 2: Suppose that G is a synchronization graph that contains exactly two SCCs, F is the set of feedforward edges in G, and F' is a resynchronization of G. Then for each $e \in F$, there exists $e' \in F'$ such that $(src(e'), snk(e'))$ subsumes e in G.

An intuitive correspondence between the pairwise resynchronization problem and the set covering problem can be derived from Fact 2. Suppose that G is a synchronization graph with exactly two SCCs C_1 and C_2 such that each feedforward edge is directed from a member of C_1 to a member of C_2. We start by viewing the set F of feedforward edges in G as the finite set that we wish to cover, and with each member p of $\{(x, y) | (x \in C_1, y \in C_2)\}$, we associate the subset of F defined by $\chi(p) \equiv \{e \in F | (p \; subsumes \; e)\}$. Thus, $\chi(p)$ is the set of feedforward edges of G whose corresponding synchronizations can be eliminated if we implement a zero-delay synchronization edge directed from the first vertex of the ordered pair p to the second vertex of p. Clearly then, $\{e_1', e_2', \ldots, e_n'\}$ is a resynchronization if and only if each $e \in F$ is contained in at least one $\chi((src(e_i'), snk(e_i')))$ — that is, if and only if $\{\chi((src(e_i'), snk(e_i'))) | 1 \le i \le n\}$ covers F. Thus, solving the pairwise resynchronization problem for G is equivalent to finding a minimal cover for F given the family of subsets $\{\chi(x, y) | (x \in C_1, y \in C_2)\}$.

Figure 3 helps to illustrate this intuition. Suppose that we are given the set

$X = \{x_1, x_2, x_3, x_4\}$, and the family of subsets $T = \{t_1, t_2, t_3\}$, where $t_1 = \{x_1, x_3\}$, $t_2 = \{x_1, x_2\}$, and $t_3 = \{x_2, x_4\}$. To construct an instance of the pairwise resynchronization problem, we first create two vertices and an edge directed between these vertices *for each* member of X; we label each of the edges created in this step with the corresponding member of X. Then for each $t \in T$, we create two vertices $vsrc(t)$ and $vsnk(t)$. Next, for each relation $x_i \in t_j$ (there are six such relations in this example), we create two delayless edges — one directed from the source of the edge corresponding to x_i and directed to $vsrc(t_j)$, and another directed from $vsnk(t_j)$ to the sink of the edge corresponding to x_i. This last step has the effect of making each pair $(vsrc(t_i), vsnk(t_i))$ subsume exactly those edges that correspond to members of t_i; in other



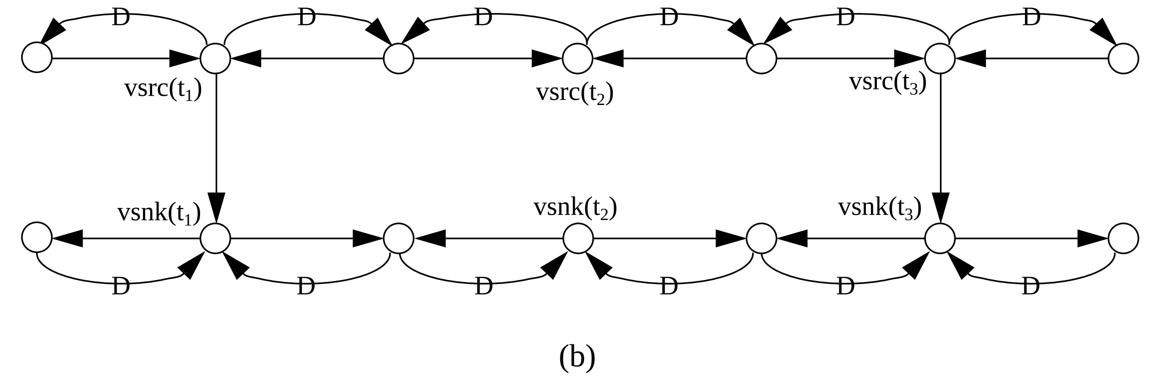


Figure 3. (a) An instance of the pairwise resynchronization problem that is derived from an instance of the set covering problem; (b) the DFG that results from a solution to this instance of pairwise resynchronization.

words, after this construction, $\chi((vsrc(t_i), vsnk(t_i))) = t_i$, for each i. Finally, for each edge created in the previous step, we create a corresponding feedback edge oriented in the opposite direction, and having a unit delay.

Figure 3(a) shows the synchronization graph that results from this construction process. Here, it is assumed that each vertex corresponds to a separate processor; the associated unit delay, self loop edges are not shown to avoid excessive clutter. Observe that the graph contains two SCCs — the SCC $(\{src(x_i)\} \cup \{vsrc(t_i)\})$ and the SCC $(\{snk(x_i)\} \cup \{vsnk(t_i)\})$ — and that the set of feedforward edges is the set of edges that correspond to members of X. Now, recall that a major correspondence between the given instance of set covering and the instance of pairwise resynchronization defined by Figure 3(a) is that $\chi((vsrc(t_i), vsnk(t_i))) = t_i$, for each i. Thus, if we can find a minimal resynchronization of Figure 3(a) such that each edge in this resynchronization is directed from some $vsrc(t_k)$ to the corresponding $vsnk(t_k)$, then the associated t_k's form a minimum cover of X. For example, it is easy, albeit tedious, to verify that the resynchronization illustrated in Figure 3(b), $\{d_0(vsrc(t_1), vsnk(t_1)), d_0(vsrc(t_3), vsnk(t_3))\}$, is a minimal resynchronization of Figure 3(a), and from this, we can conclude that $\{t_1, t_3\}$ is a minimal cover for X. From inspection of the given sets X and T, it is easily verified that this conclusion is correct.

This example illustrates how an instance of pairwise resynchronization can be constructed (in polynomial time) from an instance of set covering, and how a solution to this instance of pairwise resynchronization can easily be converted into a solution of the set covering instance. Our formal proof of the NP-hardness of pairwise resynchronization, presented in the following section, is a generalization of the example in Figure 3.

9. Intractability of resynchronization

In this section, we establish the NP completeness of the resynchronization problem, which was defined in Section 8. We establish this by reducing an arbitrary instance of the set-covering problem, a well-known NP-hard problem, to an instance of the pairwise resynchronization problem, which is a special case of the resynchronization problem that occurs when there are exactly two SCCs. The intuition behind this reduction is explained in Section 8 above.

Suppose that we are given an instance (X, T) of set covering, where X is a finite set, and T is a family of subsets of X that covers X. Without loss of generality, we assume that

$$T \text{ does } \textit{not} \text{ contain a proper nonempty subset } T' \text{ that satisfies } \left(\bigcup_{t \in (T - T')} t\right) \cap \left(\bigcup_{t \in T'} t\right) = \varnothing. \quad (11)$$

We can assume this without loss of generality because if this assumption does not hold, then we can apply the construction below to each “independent subfamily” separately, and then combine the results to get a minimal cover for X.

The following steps specify how we construct a DFG from (X, T). Except where stated otherwise, no delay is placed on the edges that are instantiated.

1. For each $x \in X$, instantiate two vertices $vsrc(x)$ and $vsnk(x)$, and instantiate an edge $e(x)$ directed from $vsrc(x)$ to $vsnk(x)$.
2. For each $t \in T$
    (a). Instantiate two vertices $vsrc(t)$ and $vsnk(t)$.
    (b). For each $x \in t$
        - Instantiate an edge directed from $vsrc(x)$ to $vsrc(t)$.
        - Instantiate an edge directed from $vsrc(t)$ to $vsrc(x)$, and place one delay on this edge.
        - Instantiate an edge directed from $vsnk(t)$ to $vsnk(x)$.
        - Instantiate an edge directed from $vsnk(x)$ to $vsnk(t)$, and place one delay on this edge.
3. For each vertex v that has been instantiated, instantiate an edge directed from v to itself, and place one delay on this edge.

Observe from our construction, that whenever $x \in X$ is contained in $t \in T$, there is an edge directed from $vsrc(x)$ $(vsnk(t))$ to $vsrc(t)$ $(vsnk(x))$, and there is also an edge (having unit delay) directed from $vsrc(t)$ $(vsnk(x))$ to $vsrc(x)$ $(vsnk(t))$. Thus, from the assumption stated in (11), it follows that $\{vsrc(z) | z \in (X \cup T)\}$ forms one SCC, $\{vsnk(z) | z \in (X \cup T)\}$ forms another SCC, and $F \equiv \{e(x) | x \in X\}$ is the set of feedforward edges.

Let G denote the DFG that we have constructed, and as in Section 8, define $\chi(p) \equiv \{e \in F | (p \textit{ subsumes } (src(e), snk(e)))\}$ for each ordered pair of vertices $p = (y_1, y_2)$ such that y_1 is contained in the source SCC of G, and y_2 is contained in the sink SCC of G. Clearly, G gives an instance of the pairwise resynchronization problem.

Observation 1: By construction of G, observe that $\{x \in X | ((vsrc(t), vsnk(t)) \textit{ subsumes } (vsrc(x), vsnk(x)))\} = t$, for all $t \in T$. Thus, for all $t \in T$, $\chi(vsrc(t), vsnk(t)) = \{e(x) | x \in t\}$.

Observation 2: For each $x \in X$, all input edges of $vsrc(x)$ have unit delay on them. It follows that for any vertex y in the sink SCC of G, $\chi(vsrc(x), y) \subseteq \{e \in F | src(e) = vsrc(x)\} = \{e(x)\}$.

Observation 3: For each $t \in T$, the only vertices in G that have a delay-free path to $vsrc(t)$ are those vertices contained in $\{vsrc(x) | x \in t\}$. It follows that for any vertex y in the sink SCC of G, $\chi(vsrc(t), y) \subseteq \chi(vsrc(t), vsnk(t)) = \{e(x) | x \in t\}$.

Now suppose that $F' = \{f_1, f_2, \ldots, f_m\}$ is a minimal resynchronization of G. For each $i \in \{1, 2, \ldots, m\}$, exactly one of the following two cases must apply

Case 1: $vsrc(f_i) = vsrc(x)$ for some $x \in X$. In this case, we pick an arbitrary $t \in T$ that contains x, and we set $v_i = vsrc(t)$ and $w_i = vsnk(t)$. From Observation 2, it follows that $\chi((src(f_i), snk(f_i))) \subseteq \{e(x)\} \subseteq \chi(v_i, w_i)$.

Case 2: $vsrc(f_i) = vsrc(t)$ for some $t \in T$. We set $v_i = vsrc(t)$ and $w_i = vsnk(t)$. From Observation 3, we have $\chi((src(f_i), snk(f_i))) \subseteq \chi(v_i, w_i)$.

Observation 4: From our definition of the v_is and w_is, $\{d_o(v_i, w_i) | (i \in \{1, 2, \ldots, m\})\}$ is a minimal resynchronization of G. Also, each (v_i, w_i) is of the form $(vsrc(t), vsnk(t))$, where $t \in T$.

Now, for each $i \in \{1, 2, \ldots, m\}$, we define

$$Z_i \equiv \{x \in X \mid (v_i, w_i) \textit{ subsumes } (vsrc(x), vsnk(x))\}.$$

Proposition 1: $\{Z_1, Z_2, \ldots, Z_m\}$ covers X.

Proof: From Observation 4, we have that for each Z_i, there exists a $t \in T$ such that $Z_i = \{x \in X \mid (vsrc(t), vsnk(t)) \textit{ subsumes } (vsrc(x), vsnk(x))\}$. Thus, each Z_i is a member of T. Also, since $\{d_o(v_i, w_i) \mid (i \in \{1, 2, \ldots, m\})\}$ is a resynchronization of G, each member of $\{(vsrc(x), vsnk(x)) \mid x \in X\}$ must be preserved by some (v_i, w_i), and thus each $x \in X$ must be contained in some Z_i. *QED.*

Proposition 2: $\{Z_1, Z_2, \ldots, Z_m\}$ is a minimal cover for X.

Proof: (By contraposition). Suppose there exists a cover $\{Y_1, Y_2, \ldots, Y_{m'}\}$ (among the members of T) for X, with $m' < m$. Then, each $x \in X$ is contained in some Y_j, and from Observation 1, $(vsrc(Y_j), vsnk(Y_j))$ subsumes $e(x)$. Thus, $\{(vsrc(Y_i), vsnk(Y_i)) \mid (i \in \{1, 2, \ldots, m'\})\}$ is a resynchronization of G. Since $m' < m$, it follows that $F' = \{f_1, f_2, \ldots, f_m\}$ is not a minimal resynchronization of G. *QED.*

In summary, we have shown how to convert an arbitrary instance (X, T) of the set covering problem into an instance G of the pairwise resynchronization problem, and we have shown how to convert a solution $F' = \{f_1, f_2, \ldots, f_m\}$ of this instance of pairwise resynchronization into a solution $\{Z_1, Z_2, \ldots, Z_m\}$ of (X, T). It is easily verified that all of the steps involved in deriving G from (X, T), and in deriving $\{Z_1, Z_2, \ldots, Z_m\}$ from F' can be performed in polynomial time. Thus, from the NP hardness of set covering [10], we can conclude that the pairwise resynchronization problem is NP hard.

10. Heuristic solutions

10.1 Applying set covering techniques to pairs of SCCs

A heuristic framework for the pairwise resynchronization problem emerges naturally from the relationship that we have established between set covering and pairwise resynchronization in Section 8. Given an arbitrary algorithm *COVER* that solves the set covering problem, and given an instance of pairwise resynchronization that consists of two SCCs C_1 and C_2, and a set S of feedforward synchronization edges directed from members of C_1 to members of C_2, this heuristic framework first computes the subset

$$\chi((u, v))=\{e \in S | (\rho_G(src(e), u) = 0) + (\rho_G(v, snk(e)) \leq delay(e))\}$$

for each ordered pair of actors (u, v) that is contained in the set

$$T \equiv \{(u', v') | (u' \text{ is in } C_1 \textbf{ and } v' \text{ is in } C_2)\},$$

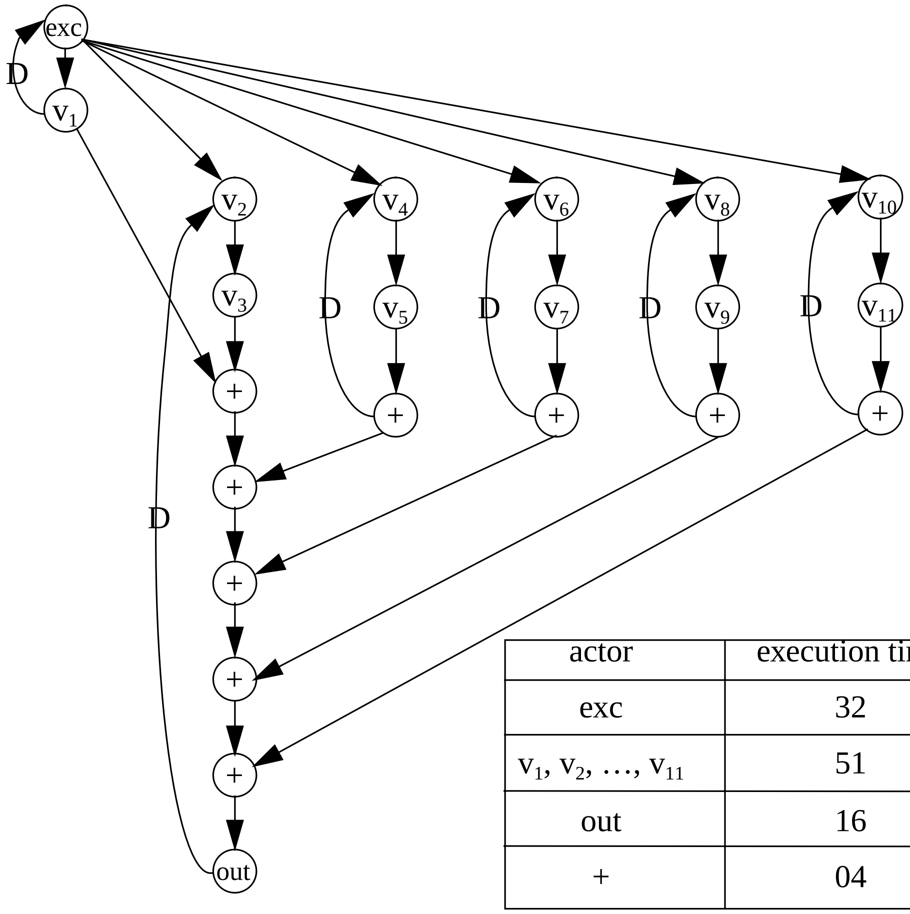
and then applies the algorithm *COVER* to the instance of set covering defined by the set S together with the family of subsets $\{\chi((u', v')) | ((u', v') \in T)\}$. If Ξ denotes the solution returned by *COVER*, then a resynchronization for the given instance of pairwise resynchronization can be derived by $\{d_0(u, v) | \chi((u, v)) \in \Xi\}$. This resynchronization is the solution returned by the heuristic framework.

From the correspondence between set covering and pairwise resynchronization that is outlined in Section 8, it follows that the quality of a resynchronization obtained by our heuristic framework is determined entirely by the quality of the solution computed by the set covering algorithm that is employed; that is, if the solution computed by *COVER* is *X*% worse (*X*% more subfamilies) than an optimal set covering solution, then the resulting resynchronization will be *X*% worse (*X*% more synchronization edges) than an optimal resynchronization of the given instance of pairwise resynchronization.

The application of our heuristic framework for pairwise resynchronization to each pair of SCCs, in some arbitrary order, in a general synchronization graph yields a heuristic framework for the general resynchronization problem. However, a major limitation of this extension to general synchronization graphs arises from its inability to consider resynchronization opportunities that involve paths that traverse more than two SCCs, and paths that contain more than one feedforward synchronization edge.

Thus, in general, the quality of the solutions obtained by this approach will be worse than the quality of the solutions that are derived by the particular set covering heuristic that is employed, and roughly, this discrepancy can be expected to increase as the number of SCCs increases relative to the number of synchronization edges in the original synchronization graph.

For example, Figure 4 shows the synchronization graph that results from a six-processor schedule of a synthesizer for plucked-string musical instruments in 11 voices based on the Karplus-Strong technique. Here, *exc* represents the excitation input, each v_i represents the computation for the ith voice, and the actors marked with “+” signs specify adders. Execution time estimates for the actors are shown in the table at the bottom of the figure. In this example, the only pair of distinct SCCs that have more than one synchronization edge between them is the pair consisting of the SCC containing $\{exc, v_1\}$ and the SCC containing v_2, v_3, five addition actors, and



actor	execution time
exc	32
$v_1, v_2, \ldots, v_{11}$	51
out	16
+	04

Figure 4. The synchronization graph that results from a six processor schedule of a music synthesizer based on the Karplus-Strong technique.

the actor labeled *out*. Thus, the best result that can be derived from the heuristic extension for general synchronization graphs described above is a resynchronization that optimally rearranges the synchronization edges between these two SCCs in isolation, and leaves all other synchronization edges unchanged. Such a resynchronization is illustrated in Figure 5. This synchronization graph has a total of nine synchronization edges, which is only one less than the number of synchronization edges in the original graph. In contrast, we will show in the following subsection that with a more flexible approach to resynchronization, the total synchronization cost of this example can be reduced to only five synchronization edges.

10.2 A more flexible approach

In this subsection, we present a more global approach to resynchronization, called Algorithm Global-resynchronize, which overcomes the major limitation of the pairwise approach discussed in Section 10.1. Algorithm Global-resynchronize is based on the simple greedy approximation algorithm for set covering that repeatedly selects a subset that covers the largest number of *remaining elements*, where a remaining element is an element that is not contained in any of the subsets that have already been selected. In [17, 26] it is shown that this set covering technique is guaranteed to compute a solution whose cardinality is no greater than $(\ln(|X|) + 1)$

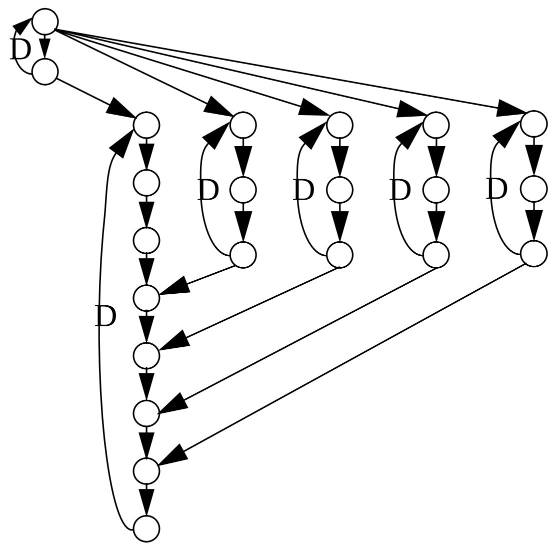


Figure 5. The synchronization graph that results from applying the heuristic framework based on pairwise resynchronization to the example of Figure 4.

times that of the optimal solution, where X is the set that is to be covered.

To adapt this set covering technique to resynchronization, we construct an instance of set covering by choosing the set X, the set of elements to be covered, to be the set of feedforward synchronization edges, and choosing the family of subsets to be

$$T \equiv \{\chi(v_1, v_2) \mid (((v_1, v_2) \notin E) \textbf{ and } (\rho_G(v_2, v_1) = \infty))\}, \tag{12}$$

where $G = (V, E)$ is the input synchronization graph. The constraint $\rho_G(v_2, v_1) = \infty$ in (12) ensures that inserting the resynchronization edge (v_1, v_2) does not introduce a cycle, and thus that it does not introduce deadlock or reduce the estimated throughput.

Algorithm Global-resynchronize assumes that the input synchronization graph is reduced (a reduced synchronization graph can be derived efficiently, for example, using the redundant synchronization removal technique presented in [5]). The algorithm determines the family of subsets specified by (12), chooses a member of this family that has maximum cardinality, inserts the corresponding delayless resynchronization edge, removes all synchronization edges that it subsumes, and updates the values $\rho_G(x, y)$ for the new synchronization graph that results. This entire process is then repeated on the new synchronization graph, and it continues until it arrives at a synchronization graph for which the computation defined by (12) produces the empty set — that is, the algorithm terminates when no more resynchronization edges can be added. Figure 6 gives a pseudocode specification of this algorithm (with some straightforward modifications to improve the running time).

To analyze the complexity of Algorithm Global-resynchronize, the following definition is useful.

Definition 4: Suppose that G is a synchronization graph.The **delayless connectivity** of G, denoted $DC(G)$, is the number of distinct ordered vertex-pairs (x, y) in G that satisfy $\rho_G(x, y) = 0$. That is,

$$DC(G) = |\hat{S}(G)|\text{, where } \hat{S}(G) = \{(x, y) | (\rho_G(x, y) = 0)\}. \tag{13}$$

The following lemma shows that as long as the input synchronization graph is reduced, the resynchronization operations performed in Algorithm Global-resynchronize always yield a reduced synchronization graph.

Lemma 3: Suppose that $G = (V, E)$ is a reduced synchronization graph; and (x, y) is an ordered pair of vertices in G such that $(x, y) \notin E$, $(\rho_G(y, x) = \infty)$, and $|\chi(x, y)| \geq 1$. Let G'

```
function Global-resynchronize
input: a reduced synchronization graph $G = (V, E)$
output: an alternative reduced synchronization graph that preserves $G$.

compute $\rho_G(x, y)$ for all actor pairs $x, y \in V$
$complete$ = FALSE
while not $(complete)$
    $best = NULL$, $M = 0$
    for $x \in V$
        for $y \in V$
            if $(\rho_G(y, x) = \infty)$ and $(x, y) \notin E$
                $\chi^* = \chi((x, y))$
                if $(|\chi^*| > M)$
                    $M = |\chi^*|$
                    $best = (x, y)$
                end if
            end if
        end for
    end for
    if $(best = NULL)$
        $complete = TRUE$
    else
        $E = E - \chi(best) + \{d_0(best)\}$
        $G = (V, E)$
        for $x, y \in V$                    /* update $\rho_G$ */
            $\rho_{new}(x, y) = min(\{\rho_G(x, y), \rho_G(x, src(best)) + \rho_G(snk(best), y)\})$
        end for
        $\rho_G = \rho_{new}$
    end if
end while
return $G$
end function
```

Figure 6. A heuristic for resynchronization.

denote the synchronization graph obtained by inserting $d_0(x, y)$ into G and removing all members of $\chi(x, y)$; that is, $G' = (V, E')$, where $E' = (E - \chi(x, y)) + \{d_0(x, y)\}$. Then G' is a reduced synchronization graph. In other words, G' does not contain any redundant synchronizations. Furthermore, $DC(G') > DC(G)$.

Proof: We prove the first part of this lemma by contraposition. Suppose that there exists a redundant synchronization edge s in G', and first suppose that $s = (x, y)$. Then from Fact 1, there exists a path in G' directed from x to y such that $Delay(p) = 0$, and

$$p \text{ does not contain } (x, y). \tag{14}$$

Also, observe that from the definition of E',

$$(E' - (x, y)) \subseteq E, \tag{15}$$

It follows from (14) and (15) that G also contains the path p.

Now let (x', y') be an arbitrary member of $\chi(x, y)$. Then

$$\rho_G(x', x) + \rho_G(y, y') \le delay(x', y'). \tag{16}$$

Since G contains the path p, we have $\rho_G(x, y) = 0$, and thus, from the triangle inequality (2) together with (16),

$$\rho_G(x', y') \le \rho_G(x', x) + \rho_G(x, y) + \rho_G(y, y') \le delay(x', y'). \tag{17}$$

We conclude that (x', y') is redundant in G, which violates the assumption that G is reduced.

If on the other hand $s \ne (x, y)$, then from Fact 1, there exists a simple path $p_s \ne (s)$ in G' directed from $src(s)$ to $snk(s)$ such that

$$Delay(p_s) \le delay(s). \tag{18}$$

Also, it follows from (15) that G contains s. Since G is reduced, the path p_s must contain the edge (x, y) (otherwise s would be redundant in G). Thus, p_s can be expressed as a concatenation

$p_s = \langle(p_1, ((x, y)), p_2)\rangle$, where either p_1 or p_2 may be empty, but not both. Furthermore, since p_s is a simple path, neither p_1 nor p_2 contains (x, y). Hence, from (15), we are guaranteed that both p_1 and p_2 are also contained in G.

Now from (18), we have

$$Delay(p_1) + Delay(p_2) \le delay(s). \tag{19}$$

Furthermore, from the definition of p_1 and p_2,

$$\rho_G(src(s), x) \le Delay(p_1) \text{ and } \rho_G(y, snk(s)) \le Delay(p_2). \tag{20}$$

Combining (19) and (20) yields

$$\rho_G(src(s), x) + \rho_G(y, snk(s)) \le delay(s), \tag{21}$$

which implies that $s \in \chi(x, y)$. But this violates the assumption that G' does not contain any edges that are subsumed by (x, y) in G. This concludes the proof of the first part of Lemma 3.

It remains to be shown that $DC(G') > DC(G)$. Now, from Lemma 1 and Definition 4, it follows that

$$\hat{S}(G) \subseteq \hat{S}(G'). \tag{22}$$

Also, from the first part of Lemma 3, which has already been proven, we know that G' is reduced. Thus, from Lemma 2, we have

$$(x, y) \notin \hat{S}(G). \tag{23}$$

But, clearly from the construction of G', $\rho_{G'}(x, y) = 0$, and thus,

$$(x, y) \in \hat{S}(G'). \tag{24}$$

From, (22), (23) and (24), it follows that $\hat{S}(G)$ is a proper subset of $\hat{S}(G')$. Hence, $DC(G') > DC(G)$. QED.

Clearly from Lemma 3, each time a Algorithm Global-resynchronize performs a resynchronization operation (an iteration of the **while** loop of Figure 6), the number of ordered vertex pairs (x, y) that satisfy $\rho_G(x, y) = 0$ is increased by at least one. Thus, the number of iterations of the **while** loop in Figure 6 is bounded above by $|V|^2$. The complexity of one iteration of the **while** loop is dominated by the computation in the pair of nested **for** loops. The computation of one iteration of the inner **for** loop is dominated by the time required to compute $\chi(x, y)$ for a specific actor pair (x, y). Assuming $\rho_G(x', y')$ is available for all $x', y' \in V$, the time to compute $\chi(x, y)$ is $O(s_c)$, where s_c is the number of feedforward synchronization edges in the current synchronization graph. Since the number of feedforward synchronization edges never increases from one iteration of the **while** loop to the next, it follows that the time-complexity of the overall algorithm is $O(s|V|^4)$, where s is the number of feedforward synchronization edges in the input synchronization graph. In practice, however, the number of resynchronization steps (**while** loop iterations) is usually much lower than $|V|^2$ since the constraints on the introduction of cycles severely limit the number of resynchronization steps. Thus, our $O(s|V|^4)$ bound can be viewed as a very conservative estimate.

10.3 Unit-subsumption resynchronization edges

At first, it may seem that it only makes sense to continue the **while** loop of Algorithm Global-resynchronize as long as a resynchronization edge can be found that subsumes at least two existing synchronization edges. However, in general it may be advantageous to continue the resynchronization process even if each resynchronization candidate subsumes at most one synchronization edge. This is because although such a resynchronization candidate does not lead to an immediate reduction in synchronization cost, its insertion may lead to future resynchronization opportunities in which the number of synchronization edges can be reduced.

Figure 7 illustrates a simple example. In the synchronization graph shown in Figure 7(a), there are 5 synchronization edges, (A, B), (B, C), (D, F), (G, F), and (A, E). Self-loop edges incident to actors A, B, C, and F (each of these four actors executes on a separate processor) are

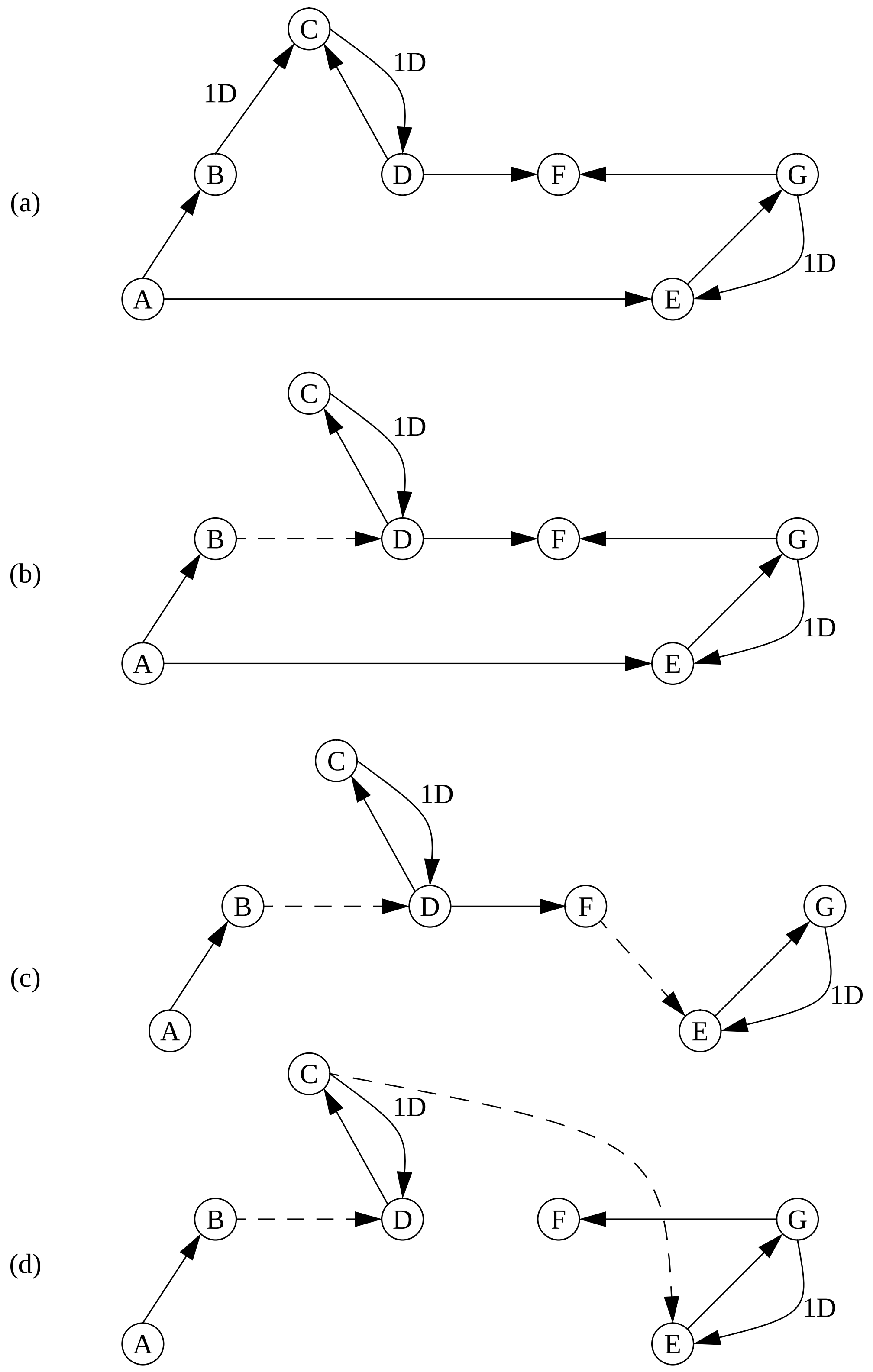


Figure 7. An example in which inserting a resynchronization edge that subsumes only one existing synchronization edge eventually leads to a reduction in the total number of synchronizations.

omitted from the illustration for clarity. It is easily verified that no resynchronization candidate in Figure 7(a) subsumes more than one synchronization edge. If we terminate the resynchronization process at this point, we must accept a synchronization cost of 5 synchronization edges.

However, suppose that we insert the resynchronization edge (B, D), which subsumes (B, C), and then we remove the subsumed edge (B, C). Then we arrive at the synchronization graph of Figure 7(c). In this graph, resynchronization candidates exist that subsume up to two synchronization edges each. For example, insertion of the resynchronization edge (F, E), allows us to remove synchronization edges (G, F) and (A, E). The resulting synchronization graph, shown in Figure 7(c), contains only four synchronization edges.

Alternatively, from Figure 7(b), we could insert the resynchronization edge (C, E) and remove both (D, F) and (A, E). This gives us the synchronization graph of Figure 7(d), which also contains four synchronization edges. This is the solution derived by our implementation of Algorithm Global-resynchronize when it is applied to the graph of Figure 7(a).

10.4 Example

Figure 8 shows the optimized synchronization graph that is obtained when Algorithm Glo-

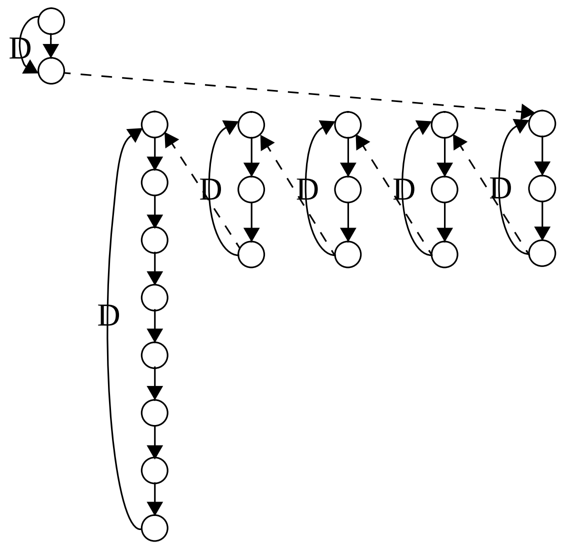


Figure 8. The optimized synchronization graph that is obtained when Algorithm *Global-resynchronize* is applied to the example of the Figure 4.

bal-resynchronize is applied to the example of Figure 4. Observe that the total number of synchronization edges has been reduced from 10 to 5. The total number of "resynchronization steps" (number of while-loop iterations) required by the heuristic to complete this resynchronization is 7.

Table 1 shows the relative throughput improvement delivered by the optimized synchronization graph of Figure 8 over the original synchronization graph as the shared memory access time varies from 1 to 10 processor clock cycles. The assumed synchronization protocol is FFS, and the back-off time for each simulation is obtained by the experimental procedure mentioned in Section 4.2. The second and fourth columns show the *average iteration period* for the original synchronization graph and the resynchronized graph, respectively. The average iteration period, which is the reciprocal of the average throughput, is the average number of time units required to execute an iteration of the synchronization graph. From the sixth column, we see that the resynchronized graph consistently attains a throughput improvement of 22% to 26%. This improvement includes the effect of reduced overhead for maintaining synchronization variables and

Memory access time	Original graph		Resynchronized graph		Percentage decrease in iter. period
	Iter. period	Memory accesses/pd	Iter. period	Memory accesses/pd	
1	250	67	195	43	22%
2	292	66	216	43	26%
3	335	64	249	43	26%
4	368	63	273	40	26%
5	408	63	318	43	22%
6	459	63	350	43	24%
7	496	63	385	43	22%
8	540	63	420	43	22%
9	584	63	455	43	22%
10	655	65	496	43	24%

Table 1. Performance comparison between the resynchronized solution and the original synchronization graph for the example of Figure 4.

reduced contention for shared memory. The third and fifth columns of Table 1 show the average number of shared memory accesses per iteration of the synchronization graph. Here we see that the resynchronized solution consistently obtains at least a 30% improvement over the original synchronization graph. Since accesses to shared memory typically require significant amounts of energy, particularly for a multiprocessor system that is not integrated on a single chip, this reduction in the average rate of shared memory accesses is especially useful when low power consumption is an important implementation issue.

10.5 Simulation approach

The simulation is written in C making use of a package called CSIM that allows concurrently running processes to be modeled. Each CSIM process is “created,” after which it runs concurrently with the other processes in the simulation. Processes communicate and synchronize through *events* and *mailboxes* (which are FIFO queues of events between two processes). Time delays are specified by the function *hold*. Holding for an appropriate time causes the process to be put into an event queue, and the process “wakes up” when the simulation time has advanced by the amount specified by the hold statement. Passage of time is modeled in this fashion. In addition, CSIM allows specification of *facilities*, which can be accessed by only one process at a time. Mutual exclusion of access to shared resources is modeled in this fashion.

For the multiprocessor simulation, each processor is made into a process, and synchronization is attained by sending and receiving messages from mailboxes. The shared bus is made into a facility. Polling of the mailbox for checking the presence of data is done by first reserving the bus, then checking for the message count on that particular mailbox; if the count is greater than zero, data can be read from shared memory, or else the processor backs off for a certain duration, and then resumes polling.

When a processor sends data, it increments a counter in shared memory, and then writes the data value. When a processor receives, it first polls the corresponding counter, and if the counter is non-zero, it proceeds with the read; otherwise, it backs off for some time and then polls the counter again. We used experimentally determined back-off times for each value of the memory access time. For a send, the processor checks if the corresponding buffer is full or not. For the simulation, all buffers are sized equal to 5; these sizes can of course be jointly minimized to reduce buffer memory. Polling time is defined as the time required to access the bus and check the counter value.

11. Efficient, optimal resynchronization for a class of synchronization graphs

In this section, we show that although optimal resynchronization is intractable for general synchronization graphs, a broad class of synchronization graphs exists for which optimal resynchronizations can be computed using an efficient polynominal-time algorithm.

11.1 Chainable synchronization graph SCCs

Definition 5: Suppose that C is an SCC in a synchronization graph G, and x is an actor in C. Then x is an **input hub** of C if for each feedforward synchronization edge e in G whose sink actor is in C, we have $\rho_C(x, snk(e)) = 0$. Similarly, x is an **output hub** of C if for each feedforward synchronization edge e in G whose source actor is in C, we have $\rho_C(src(e), x) = 0$. We say that C is **linkable** if there exist actors x, y in C such that x is an input hub, y is an output hub, and $\rho_C(x, y) = 0$. A synchronization graph is **chainable** if each SCC is linkable.

For example, consider the SCC in Figure 9(a), and assume that the dashed edges represent the synchronization edges that connect this SCC with other SCCs. This SCC has exactly one input hub, actor A, and exactly one output hub, actor F, and since $\rho(A, F) = 0$, it follows that the SCC is linkable. However, if we remove the edge (C, F), then the resulting graph (shown in Figure 9(b)) is not linkable since it does not have an output hub. A class of linkable SCCs that occur commonly in practical synchronization graphs are those SCCs that correspond to only one processor, such as the SCC shown in Figure 9(c). In such cases, the first actor executed on the processor is always an input hub and the last actor executed is always an output hub.

In the remainder of this section, we assume that for each linkable SCC, an input hub x and output hub y are selected such that $\rho(x, y) = 0$, and these actors are referred to as the **selected input hub** and the **selected output hub** of the associated SCC. Which input hub and output hub are chosen as the “selected” ones makes no difference to our discussion of the techniques in this section as long they are selected so that $\rho(x, y) = 0$.

An important property of linkable synchronization graphs is that if C_1 and C_2 are distinct linkable SCCs, then all synchronization edges directed from C_1 to C_2 are subsumed by the single ordered pair (l_1, l_2), where l_1 denotes the selected output hub of C_1 and l_2 denotes the selected input hub of C_2. Furthermore, if there exists a path between two SCCs C_1', C_2' of the form $((o_1, i_2), (o_2, i_3), \ldots, (o_{n-1}, i_n))$, where o_1 is the selected output hub of C_1', i_n is the selected input hub of C_2', and there exist distinct SCCs $Z_1, Z_2, \ldots, Z_{n-2} \notin \{C_1', C_2'\}$ such that for $k = 2, 3, \ldots, (n-1)$, i_k, o_k are respectively the selected input hub and the selected output hub of Z_{k-1}, then all synchronization edges between C_1' and C_2' are redundant.

From these properties, an optimal resynchronization for a chainable synchronization graph can be constructed efficiently by computing a topological sort of the SCCs, instantiating a zero delay synchronization edge from the selected output hub of the ith SCC in the topological sort to the selected input hub of the $(i+1)$th SCC, for $i = 1, 2, \ldots, (n-1)$, where n is the total number

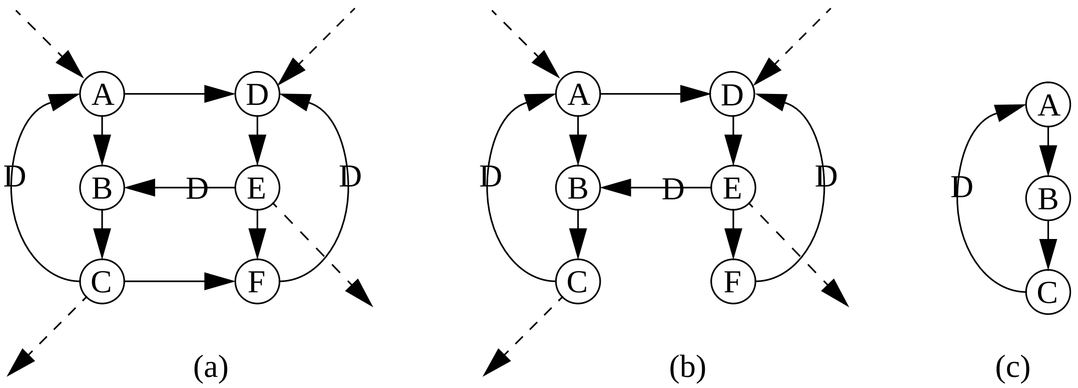


Figure 9. An illustration of input and output hubs for synchronization graph SCCs.

of SCCs, and then removing all of the redundant synchronization edges that result. For example, if this algorithm is applied to the chainable synchronization graph of Figure 10(a), then the synchronization graph of Figure 10(b) is obtained, and the number of synchronization edges is reduced from 4 to 2.

This chaining technique can be viewed as a form of pipelining, where each SCC in the output synchronization graph corresponds to a pipeline stage. Pipelining has been used extensively to increase throughput via improved parallelism ("temporal parallelism") in multiprocessor DSP implementations (see for example, [2, 15, 28]). However, in our application of pipelining, the load of each processor is unchanged, and the estimated throughput is not affected (since no new cyclic paths are introduced), and thus, the benefit to the *overall* throughput of our chaining technique arises chiefly from the optimal reduction of synchronization overhead.

The time-complexity of our optimal algorithm for resychronizing chainable synchronization graphs is $O(v^2)$, where v is the number of synchronization graph actors.

11.2 Comparison to the Global-Resynchronize heuristic

It is easily verified that the original synchronization graph for the music synthesis example of Section 10.2, shown in Figure 4, is chainable. Thus, the chaining technique presented in Sec-

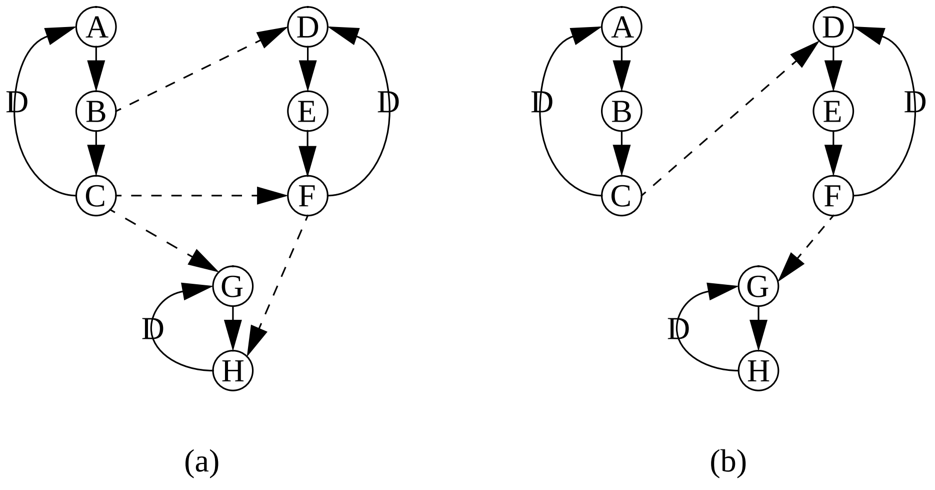


Figure 10. An illustration of an algorithm for optimal resynchronization of chainable synchronization graphs. The dashed edges are synchronization edges.

tion 11.1 is guaranteed to produce an optimal resynchronization for this example, and since no feedback synchronization edges are present, the number of synchronization edges in the resynchronized solution is guaranteed to be equal to one less than the number of SCCs in the original synchronization graph; that is, the optimized synchronization graph contains $6 - 1 = 5$ synchronization edges. From Figure 8, we see that this is precisely the number of synchronization edges in the synchronization graph that results from our implementation of Algorithm Global-resynchronize.

However, Algorithm Global-resynchronize does not always produce optimal results for chainable synchronization graphs. For example, consider the synchronization graph shown in Figure 11(a), which corresponds to an eight-processor schedule in which each of the following subsets of actors are assigned to a separate processor — $\{I\}$, $\{J\}$, $\{G, K\}$, $\{C, H\}$, $\{D\}$, $\{E, L\}$, $\{A, F\}$, and $\{B\}$. The dashed edges are synchronization edges, and the remaining edges connect actors that are assigned to the same processor. The total number of synchronization edges is 14. Now it is easily verified that actor K is both an input hub and an output hub for the SCC $\{C, G, H, J, K\}$, and similarly, actor L is both an input and output hub for the SCC $\{A, D, E, F, L\}$. Thus, we see that the overall synchronization graph is chainable. It is easily verified that the chaining technique developed in Section 11.1 uniquely yields the optimal resynchronization illustrated in Figure 11(b), which contains only 11 synchronization edges.

In contrast, the quality of the resynchronization obtained for Figure 11(a) by Algorithm Global-resynchronize depends on the order in which the actors are traversed by each of the two nested **for** loops in Figure 6. For example, if both loops traverse the actors in alphabetical order, then Global-resynchronize obtains the sub-optimal solution shown in Figure 11(c), which contains 12 synchronization edges.

However, actor traversal orders exist for which Global-resynchronize achieves optimal resynchronizations of Figure 11(a). One such ordering is $K, D, C, B, E, F, G, H, I, J, L, A$; if both **for** loops traverse the actors in this order, then Global-resynchronize yields the same resynchronized graph that is computed uniquely by the chaining technique of Section 11.1 (Figure 11(b)). It is an open question whether or not given an arbitrary chainable synchronization graph, actor traversal orders always exist with which Global-resynchronize arrives at optimal resynchronizations. Furthermore, even if such traversal orders are always guaranteed to exist, it is doubtful that they can, in general, be computed efficiently.

11.3 A generalization of the chaining technique

The chaining technique developed in Section 11.1 can be generalized to optimally resynchronize a somewhat broader class of synchronization graphs. This class consists of all synchronization graphs for which each source SCC has an output hub (but not necessarily an input hub), each sink SCC has an input hub (but not necessarily an output hub), and each internal SCC is linkable. In this case, the internal SCCs are pipelined as in the previous algorithm, and then for each source SCC, a synchronization edge is inserted from one of its output hubs to the selected input hub of the first SCC in the pipeline of internal SCCs, and for each sink SCC, a synchronization edge is inserted to one of its input hubs from the selected output hub of the last SCC in the pipeline of internal SCCs. If there are no internal SCCs, then the sink SCCs are pipelined by selecting one input hub from each SCC, and joining these input hubs with a chain of synchronization edges. Then a synchronization edge is inserted from an output hub of each source SCC to an input hub of the first SCC in the chain of sink SCCs.

11.4 Incorporating the chaining technique

In addition to guaranteed optimality, another important advantage of the chaining technique for chainable synchronization graphs is its relatively low time-complexity ($O(v^2)$ versus $O(sv^4)$ for Global-resynchronize), where v is the number of synchronization graph actors, and s is the number of feedforward synchronization edges. The primary disadvantage is, of course, its restricted applicability. An obvious solution is to first check if the general form of the chaining technique (described above in Section 11.3) can be applied, apply the chaining technique if the check returns an affirmative result, or apply Algorithm Global-resynchronize if the check returns a negative result. The check must determine whether or not each source SCC has an output hub, each sink SCC has an input hub, and each internal SCC is linkable. This check can be performed in $O(n^3)$ time, where n is the number of actors in the input synchronization graph, using a straightforward algorithm.

A useful direction for further investigation is a deeper integration of the chaining tech-

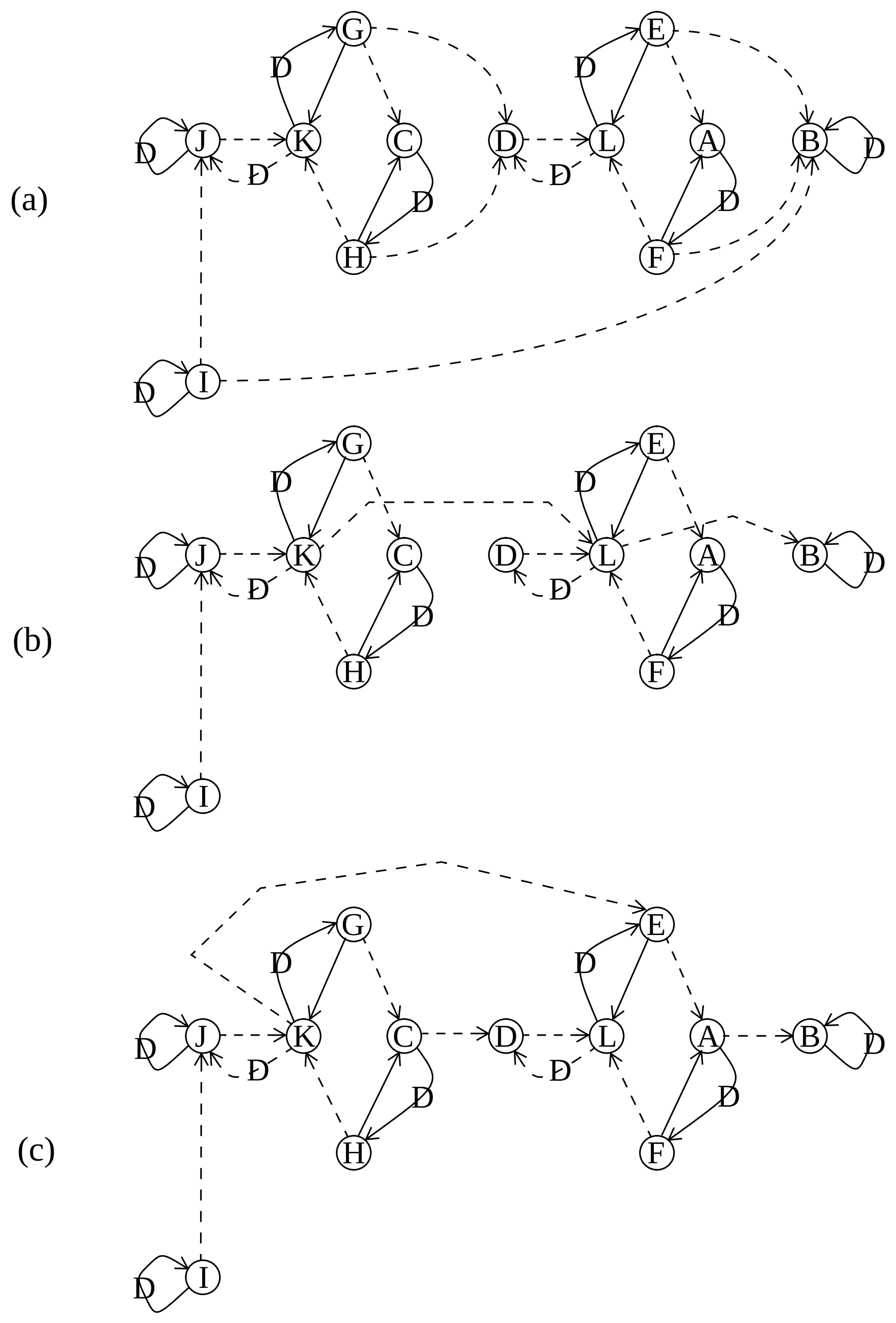


Figure 11. A chainable synchronization graph for which Algorithm Global-resynchronize fails to produce an optimal solution.

nique with algorithm Global-resynchronize for general (not necessarily chainable) synchronization graphs.

12. Conclusions

This paper develops a post-optimization called resynchronization for self-timed, multiprocessor implementations of DSP algorithms. The goal of resynchronization is to introduce new synchronizations in such a way that the number of additional synchronizations that become redundant exceeds the number of new synchronizations that are added, and thus the net synchronization cost is reduced.

We show that optimal resynchronization is intractable by deriving a reduction from the classic set covering problem. However, we define a broad class of systems for which optimal resynchronization can be performed in polynomial time. We also present a heuristic algorithm for resynchronization of general systems that emerges naturally from the correspondence to set covering. We illustrate the performance of our implementation of this heuristic on a multiprocessor schedule for a music synthesis system. The results demonstrate that the heuristic can efficiently reduce synchronization overhead and improve throughput significantly.

Several useful directions for further work emerge from our study. These include investigating whether efficient techniques can be developed that consider resynchronization opportunities within strongly connected components, rather than just across feedforward edges. There may also be considerable room for improvement over our proposed heuristic, which is a straightforward adaptation of an existing set covering algorithm. In particular, it would be useful to explore ways to best integrate the proposed heuristic for general synchronization graphs with the optimal chaining method for a restricted class of graphs, and it may be interesting to search for properties of practical synchronization graphs that could be exploited in addition to the correspondence with set covering. The extension of Sakar's concept of counting semaphores [35] to self-timed, iterative execution, and the incorporation of extended counting semaphores within our resynchronization framework are also interesting directions for further work.

13. Glossary

$\lvert S \rvert$:	The number of members in the finite set S.
$\rho(x, y)$:	Same as ρ_G with the DFG G understood from context.
$\rho_G(x, y)$:	If there is no path in G from x to y, then $\rho_G(x, y) = \infty$; otherwise, $\rho_G(x, y) = Delay(p)$, where p is any minimum-delay path from x to y.
$delay(e)$:	The delay on a DFG edge e.
$Delay(p)$:	The sum of the edge delays over all edges in the path p.
$d_n(u, v)$:	An edge whose source and sink vertices are u and v, respectively, and whose delay is equal to n.
λ_{max}:	The maximum cycle mean of a DFG.
$\chi(p)$:	The set of synchronization edges that are subsumed by the ordered pair of actors p.
$\langle(p_1, p_2, \ldots, p_k)\rangle$:	The *concatenation* of the paths $p_1, p_2, \ldots, p_k$.
critical cycle:	A cycle in a DFG whose cycle mean is equal to the maximum cycle mean of the DFG.
cycle mean:	The cycle mean of a cycle C in a DFG is equal to T/D, where T is the sum of the execution times of all vertices traversed by C, and D is the sum of delays of all edges in C.
estimated throughput:	Given a DFG with execution time estimates for the actors, the estimated throughput is the reciprocal of the maximum cycle mean.
FBS:	Feedback synchronization. A synchronization protocol that may be used for feedback edges in a synchronization graph.
feedback edge:	An edge that is contained in at least one cycle.
feedforward edge:	An edge that is not contained in a cycle.
FFS:	Feedforward synchronization. A synchronization protocol that may be used for feedforward edges in a synchronization graph.
maximum cycle mean:	Given a DFG, the maximum cycle mean is the largest cycle mean over all cycles in the DFG.
reduced	A synchronization graph is **reduced** if it does not contain any redundant synchronization edges.

resynchronization edge:	Given a synchronization graph G and a resynchronization R, a resynchronization edge of R is any member of R that is not contained in G.
$\Psi(R, G)$:	If G is a synchronization graph and R is a resynchronization of G, then $\Psi(R, G)$ denotes the graph that results from the resynchronization R.
SCC:	Strongly connected component.
self loop:	An edge whose source and sink vertices are identical.
subsumes:	Given a synchronization edge (x_1, x_2) and an ordered pair of actors (y_1, y_2), (y_1, y_2) subsumes (x_1, x_2) if $\rho(x_1, y_1) + \rho(y_2, x_2) \le delay((x_1, x_2))$.
$t(v)$:	The execution time or estimated execution time of actor v.

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